Risk ratio, odds ratio, risk difference:

Which causal measure is easier to generalize?

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Causal measure formalism

$$\mathbb{E}\left[Y^{(1)}\right] \qquad \mathbb{E}\left[Y^{(0)}\right]$$

Expected outcome if treated (1) or control (0)

Count the dead

$$\tau_{RR} = \frac{\mathbb{E}\left[Y^{(1)}\right]}{\mathbb{E}\left[Y^{(0)}\right]}$$

Count the living

$$\tau_{SR} = \frac{1-\mathbb{E}\left[Y^{(1)}\right]}{1-\mathbb{E}\left[Y^{(0)}\right]}$$

$$\tau_{RD} = \mathbb{E}\left[Y^{(1)}\right] - \mathbb{E}\left[Y^{(0)}\right]$$

Risk Difference

$$\tau_{NNT} = \tau_{RD}^{-1}$$

Number Needed to Treat

Odds Ratio

$$\tau_{OR} = \frac{\mathbb{E}[Y^{(1)}]}{1-\mathbb{E}[Y^{(1)}]} \left(\frac{\mathbb{E}[Y^{(0)}]}{1-\mathbb{E}[Y^{(0)}]}\right)^{-1}$$

A variety of measures….

E.g. benefit of antihyperintensive therapy (A) against stroke (Y) **[1]**

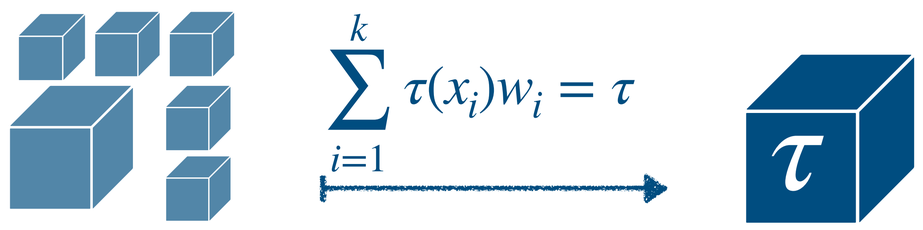
	τ_{RD}	τ_{RR}	τ_{SR}	τ_{NNT}	τ_{OR}
All (P_s)	−0.0452	**0.6**	1.05	22	0.57
X = 1	−0.006	**0.6**	1.01	167	0.6
X = 0	−0.08	**0.6**	1.1	13	0.545

👩 *``Treated group has 0.6 times the risk of having a stroke outcome when compared with the placebo." — ``The Number Needed to Treat is 22." — ``Effect is stronger on subgroup X=0 but not on the ratio scale."*

— leading to different impressions and heterogeneity patterns

A desirable property: collapsibility

i.e. population's effect is equal to a weighted sum of local effects



— Unfortunately, not all measures are collapsible (e.g. OR, log-OR, NNT)

Reverse the thinking, through the lens of baseline covariates

— Every generative process can be decomposed in a baseline level and alteration part with no assumption.

Continuous outcome — Robinson decomposition [2]

$$\mathbb{E}\left[Y^{(a)} \mid X\right] = b(X) + a\, m(X)$$

Baseline — Additivity — Modification

$$\tau_{RD}(x) = m(x) \qquad \tau_{RR}(x) = 1 + m(x)/b(x)$$

No entanglement — Entanglement

Binary outcome — What model?

If treatment increases occurrences (i.e. deleterious effect)

$$\mathbb{P}\left[Y^{(a)} = 1 \mid X\right] = b(X) + a\left(1 - b\,(X)\right) m(X)$$

Simple additivity is not possible anymore due to the binary nature of Y

$$\tau_{RD}(x) = (1 - b(x))m(x) \qquad \tau_{SR}(x) = 1 - m(x)$$

Entanglement — No entanglement

Different sets of covariates for generalization of causal effects?

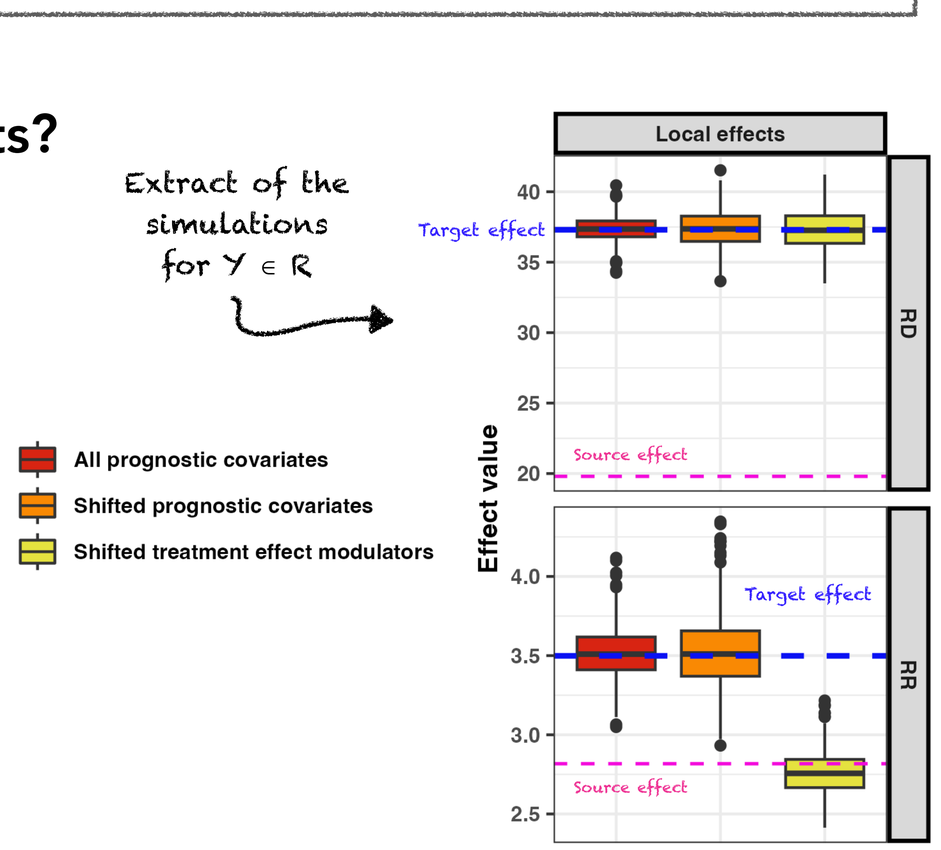
Transferring trial's findings to a target population

Generalisation: re-weighting trial local effects $\tau(x)$ [3,4,5]

— requires measure collapsibility
— requires in general all shifted covariates that are prognostic or treatment effect modifiers
— but some measures requires less covariates(*) as soon as they locally disentangle baseline and modification
—- if Y is binary RR or SR, if Y is continuous RD

(*) This may require to have access to Y(0) in the target sample. As an alternative, one can generalize conditional outcomes but with all shifted prognostic covariates

Extract of the simulations for $Y \in R$



— RD can be recovered with less covariates

[1] Cook and Sackett, 1995; [2] Robinson, 1988; [3] Pearl and Bareinboim, 2011;
[4] Huitfeld, 2018; [5] Pearl and Cinelli 2020